

**FIELDS 221/222/245: MULTI-SUBJECT: TEACHERS OF
CHILDHOOD (GRADE 1–GRADE 6)
ASSESSMENT FRAMEWORK**

FIELD 222: PART TWO: MATHEMATICS

COMPETENCY 0001—NUMBER AND OPERATIONS

Performance Expectations

The New York State Grade 1–Grade 6 Multi-Subject teacher demonstrates deep knowledge of number and operations and algebraic thinking. The teacher interprets arithmetic as a coherent and logical subject that makes sense and demonstrates understanding of how operations used for whole numbers and fractions form the basis for further work in algebra. The teacher applies operations and algebraic thinking to model and solve problems and works accurately with numbers and algebraic expressions and equations. The teacher interprets numbers and the base-ten system as a coherent and logical set of ideas; extends the properties of whole numbers and number operations to fractions; and analyzes properties of fractions, decimals, and percents. The teacher applies understanding of place value and properties of operations to justify algorithms; works accurately with whole numbers, fractions, decimals, and percents; and uses numbers and operations to model and solve mathematical and real-world problems.

1.1 Operations and Algebraic Thinking

Performance Indicators

- a. applies operations and relationships between operations (e.g., division as an unknown factor problem)
- b. analyzes properties of factors and multiples
- c. applies strategies for writing and interpreting numerical expressions
- d. generates and analyzes patterns and relationships and identifies apparent features of patterns that are not explicit in the rule used to generate them
- e. applies and extends principles of arithmetic and the order of operations to algebraic expressions, equations, and inequalities
- f. uses properties of operations to generate equivalent expressions
- g. analyzes and solves linear equations and inequalities and pairs of simultaneous linear equations
- h. solves mathematical and real-world problems using numerical and algebraic expressions and equations

1.2 Number and Operations—Base Ten and Fractions

Performance Indicators

- a. demonstrates knowledge of place value
- b. applies understanding of place value and properties of operations to round, add, subtract, multiply, and divide multidigit numbers

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- c. analyzes decimal notation and compares decimals, decimal fractions, and fractions
- d. justifies computational algorithms
- e. analyzes and performs operations with decimals
- f. applies number theory concepts (e.g., primes, divisibility, factors, least common multiple, greatest common factor)
- g. extends number operations to fractions and performs operations on fractions
- h. applies properties of signed rational numbers, ordering, and the absolute value of rational numbers
- i. applies and extends understanding of operations with fractions to add, subtract, multiply, and divide rational numbers
- j. solves mathematical and real-world problems involving the four basic operations with rational numbers, including the use of the distributive law to justify properties of rational numbers

**COMPETENCY 0002—RATIOS AND PROPORTIONAL RELATIONSHIPS AND
NUMBER SYSTEMS**

Performance Expectations

The New York State Grade 1–Grade 6 Multi-Subject teacher demonstrates deep knowledge of ratios and proportional relationships. The teacher applies connections between multiplication and division and ratios and rates, as well as connections between ratios and proportional reasoning, linear equations, and concepts of measurement and geometry. The teacher analyzes properties of whole, rational, and real numbers and interprets the real number system as an extension of the rational numbers. The teacher works accurately with ratios and proportional relationships and rational numbers and uses them to model and solve mathematical and real-world problems.

2.1 Ratios and Proportional Relationships

Performance Indicators

- a. solves unit rate problems, including those involving unit pricing; constant speed; and ratios of lengths, areas, and other quantities measured in like or unlike units
- b. interprets percents of a quantity as a rate per 100 and solves mathematical and real-world problems involving percents
- c. identifies the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships
- d. represents proportional relationships by equations

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- e. explains and analyzes the relationships between graphs of proportional relationships in terms of the situation represented by the relationship
- f. uses proportional relationships to solve multistep ratio and percent problems (e.g., simple interest rates, commissions, percent increase or decrease, percent error)
- g. analyzes the connections between proportional relationships, lines, and linear equations
- h. uses similar triangles to explain why the slope is the same between any two distinct points on a nonvertical line in the coordinate plane and graphs and analyzes linear equations

2.2 Rational and Real Number Systems

Performance Indicators

- a. applies knowledge of numbers that are not rational and finds rational approximations of irrational numbers
- b. applies properties of repeating decimal expansions and converts between repeating decimal expansions and rational numbers
- c. analyzes and applies properties of integer exponents and extends them to rational exponents
- d. analyzes how the definition and meaning of rational exponents allows for extending the properties of integer exponents
- e. rewrites expressions involving radicals and rational exponents using the properties of exponents
- f. uses square roots and cube roots to represent solutions to problems and equations
- g. performs operations with numbers expressed in scientific notation
- h. uses properties of rational and irrational numbers
- i. uses units as a way to understand problems and to guide the solution of multistep problems and chooses and interprets units consistently in formulas

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COMPETENCY 0003—ALGEBRA, MEASUREMENT, GEOMETRY, AND DATA

Performance Expectations

The New York State Grade 1–Grade 6 Multi-Subject teacher demonstrates knowledge of the structure of algebraic expressions, how algebraic manipulations are governed by properties of operations and exponents, the nature of solutions to equations, and reasoning processes for manipulating expressions and solving equations. The teacher analyzes functions, uses expressions to define functions, applies properties of functions, and analyzes graphs. The teacher uses algebra to model and solve problems and demonstrates skill and accuracy in working with algebraic expressions, equations, and functions. The teacher demonstrates deep knowledge of measurement and geometry and interprets geometry as a system based on precise definitions and mathematical reasoning. The teacher works with and interprets data, uses measures of center and variability, and draws inferences from data distributions. The teacher applies knowledge of probability to analyze chance events and understands how statistics and chance processes are used to make inferences. The teacher applies measurement, geometry, and data concepts to model and solve mathematical and real-world problems.

3.1 Algebra

Performance Indicators

- a. understands the vocabulary of mathematical expressions (e.g., terms, factors, coefficients) and interprets their structure
- b. writes expressions in equivalent forms to solve problems (e.g., factor quadratic expressions, complete the square, use properties of exponents)
- c. performs arithmetic on polynomials and understands the relationship between zeros and factors of polynomials
- d. creates equations and inequalities in one variable and uses them to solve mathematical and real-world problems (e.g., based on verbal descriptions, tables, graphs), including equations that arise from linear, quadratic, and simple rational and exponential functions
- e. creates equations in two or more variables to represent relationships between quantities and analyzes graphs of equations on coordinate axes
- f. uses systems of equations or inequalities to represent situations, including constraints (e.g., uses inequalities to represent nutritional and cost constraints on combinations of different foods)
- g. analyzes solving equations as a process of reasoning, explains the reasoning, solves equations and inequalities in one variable, and solves systems of equations in two variables
- h. applies the concept of a function, identifies the range and domain of a function, and uses function notation appropriately

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- i. interprets functions that arise in applications in terms of the context and analyzes key features of functions (e.g., intercepts, intervals where the function is increasing, relative maximums or minimums, zeros, asymptotes, end behavior)
- j. analyzes functions (e.g., linear, quadratic, square root, piecewise, polynomial, exponential, logarithmic) using different representations, such as graphs, verbal descriptions, equivalent algebraic forms, and numeric tables
- k. constructs and compares linear, quadratic, and exponential models and distinguishes between those situations (mathematical and real-world) that can be modeled with linear functions and those that can be modeled with exponential functions

3.2 Measurement and Geometry

Performance Indicators

- a. analyzes attributes of shapes, including symmetry and properties of their lines and angles
- b. solves problems involving measurement and conversions of measurement units
- c. solves mathematical and real-world problems involving angle measure, perimeter, area, surface area, and volume
- d. solves problems involving congruence and analyzes congruence in terms of a sequence of transformations (i.e., rotations, reflections, and translations)
- e. graphs points and shapes on the coordinate plane to solve mathematical and real-world problems
- f. applies the Pythagorean theorem to solve a variety of problems, including distance problems in the coordinate plane
- g. solves problems involving similarity and analyzes similarity in terms of scale factors and similarity transformations

3.3 Data, Statistics, and Probability

Performance Indicators

- a. represents, analyzes, and solves problems with data presented in various forms (e.g., line plots, bar graphs, picture graphs)
- b. demonstrates knowledge of statistical variability and measures and summarizes and describes data distributions (e.g., number lines, dot plots, histograms, box plots)
- c. demonstrates knowledge of the use of random sampling to draw inferences about a population

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- d. draws informal or comparative inferences about two populations using data distributions and measures of center (e.g., mean, median) and variability (e.g., interquartile range, mean absolute deviation, standard deviation)
- e. investigates chance processes and develops, uses, and evaluates probability models (e.g., independent and dependent events)
- f. investigates patterns of association in bivariate data using scatter plots, linear models, and two-way tables

COMPETENCY 0004—INSTRUCTION IN MATHEMATICS

Performance Expectations

The New York State Grade 1–Grade 6 Multi-Subject teacher applies knowledge of how students learn number concepts, operations, and algebraic thinking; fractions and ratios; and proportional relationships. The teacher applies knowledge of how students develop measurement and spatial reasoning concepts and skills related to data collection and interpretation. The teacher provides a rich variety of focused strategies (e.g., moving from concrete to abstract; using multiple representations; explaining, connecting, and critiquing ideas) for promoting students' understanding, confidence, perseverance, and fluency in these areas. The focused strategies include explicitly teaching mathematical language that students need for mathematical practice, performance, and success. The teacher uses assessment data to differentiate instruction.

4.1 Instruction in Number and Operations and Algebraic Thinking

Performance Indicators

- a. applies strategies for teaching properties of whole numbers, counting, methods for composing and decomposing numbers, and multiple ways of representing numbers
- b. demonstrates knowledge of strategies for teaching place value concepts
- c. demonstrates knowledge of strategies that build understanding of the equal sign and the meaning of equations
- d. applies strategies for developing students' fluency with number operations
- e. applies strategies for teaching operations and the relationship between operations (e.g., division as an unknown factor problem)
- f. applies methods for teaching how to represent and solve one- and two-step problems involving addition, subtraction, multiplication, and division
- g. applies methods for teaching how to round, add, subtract, multiply, and divide multidigit numbers
- h. applies strategies for teaching and justifying computational algorithms

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- i. applies methods for extending students' understanding of numbers to the system of rational numbers, including concepts associated with ordering, absolute value, and negative numbers
- j. applies strategies for extending students' understanding of arithmetic and the order of operations to algebraic expressions
- k. applies strategies for teaching the meaning of equations and inequalities and how to solve them
- l. applies strategies for teaching how to use variables to represent and analyze relationships between dependent and independent variables

4.2 Instruction in Fractions and Ratios and Proportional Relationships

Performance Indicators

- a. applies methods for teaching how to develop understanding of fractions as numbers
- b. applies strategies for extending understanding of fraction equivalence and ordering
- c. demonstrates knowledge of strategies for teaching how to build fractions from unit fractions by applying and extending understanding of operations of whole numbers
- d. demonstrates knowledge of strategies for teaching decimal notation for fractions and for performing operations with decimals
- e. demonstrates knowledge of strategies for teaching the use of equivalent fractions as a strategy to add and subtract fractions
- f. applies strategies for teaching concepts of rate, ratio, unit rates, ratio language, and ratio relationships and for teaching connections between multiplication, division, ratio, rates, and fractions
- g. analyzes strategies for teaching the use of ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, equations, pairs of values plotted in the coordinate plane)
- h. demonstrates knowledge of strategies for teaching how to use ratio reasoning to convert measurement units
- i. applies techniques for teaching unit rate problems, including those involving unit pricing and constant speed, and for teaching ratios of lengths, areas, and other quantities measured in like or unlike units

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4.3 Instruction in Measurement and Data

Performance Indicators

- a. applies strategies for teaching how to describe and compare measurable attributes
- b. applies strategies for teaching how to classify and count objects in categories
- c. demonstrates knowledge of strategies for teaching how to measure indirectly by iterating length units and how to measure and estimate lengths in standard units
- d. applies strategies for relating addition and subtraction to length and for relating multiplication and addition to area
- e. applies strategies for teaching how to compare, create, and compose shapes and how to analyze attributes of shapes, including symmetry and properties of their lines and angles
- f. selects strategies for teaching how to tell and write time and work with money
- g. applies strategies for teaching how to classify objects and generate and represent measurement data
- h. applies strategies for teaching concepts of perimeter, area, and volume and their relationships to number operations
- i. applies strategies for teaching how to generate and represent measurement data and to solve problems with data (e.g., using line plots, bar graphs, or picture graphs)
- j. applies strategies for developing understanding of statistical concepts (e.g., statistical variability, data collection, measures of center, shapes of data distributions)

COMPETENCY 0005—ANALYSIS, SYNTHESIS, AND APPLICATION

Performance Expectations

The New York State Grade 1–Grade 6 Multi-Subject teacher accurately and effectively applies relevant content knowledge and pedagogical content knowledge in number and operations, operations and algebraic thinking, fractions, ratios and proportional reasoning, and measurement and data to analyze and synthesize assessment data about an individual student, identify conceptual or procedural errors, and provide a well-reasoned and accurate analysis of the student's mathematical knowledge. The teacher uses the assessment results and knowledge of how students learn to present an appropriate instructional approach that meets the needs of the student.

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- a. analyzes and interprets samples of a student's work and other assessment data to monitor student progress and determine a student's strengths and areas of need in mathematics
- b. demonstrates knowledge of the content by identifying and analyzing any errors or misconceptions in work samples
- c. describes appropriate and effective content-specific instructional strategies, activities, or interventions to address a student's identified needs
- d. demonstrates the ability to generate real-world scenarios that illustrate specific mathematical concepts
- e. demonstrates the ability to justify the effectiveness of selected instructional strategies, activities, or interventions for promoting a student's mathematical understanding